INDIAN SCHOOL MUSCAT

DEPARTMENT OF MATHEMATICS

POWER CAPSULE Unit -1

RELATIONS AND FUNCTIONS

CLASS 12

1. Write the principal value of

(i) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ **(ii)** $\cos^{-1}(\sqrt{3}/2)$ **(iii)** $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ **(iv)** $\sec^{-1}(-2)$.

2. What is the value of the following functions (using principal value)

(i) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) - \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ **(ii)** $\sin^{-1}\left(-\frac{1}{2}\right) - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(iii) $\sin\left\{\frac{\pi}{6} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right\}$ **(iv)** $\tan\left(\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right)$

3. If $\tan^{-1} x + \tan^{-1} y = \frac{4\pi}{5}$, find $\cot^{-1} x + \cot^{-1} y$.

4. Principal value of $\sin^{-1} 1/2$ is

(a) $\frac{\pi}{3}$ (b) $-\frac{\pi}{3}$

(c) $\frac{5\pi}{6}$ (d) $-\frac{\pi}{6}$

5. $\tan^{-1}\{\sin(-\frac{\pi}{2})\}$ is equal to

(a) -1

(b) 1

(c) $\frac{\pi}{2}$

(d) $-\frac{\pi}{4}$

6. Principal value of the expression $\cos^{-1}[\cos(-680°)]$ is

(a) $\frac{2\pi}{9}$ (b) $-\frac{2\pi}{9}$

(c) $\frac{34\pi}{9}$ (d) $\frac{\pi}{9}$

7. . The value of $\tan^2(\sec^{-1}2) + \cot2(\text{cosec}^{-1}3)$ is

(a) 5

(b) 11

(c) 13

(d) 15

8. Find the values of the following

   1. $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$
   2. $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$
   3. $\tan^{-1}(\sqrt{3}) + \sec^{-1}(-2) + \text{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$

9. $4\sin^{-1}x + \cos^{-1}x = \pi$, then find the value of x.

10. If $3\tan^{-1}x + \cot^{-1}x = \pi$ then find x

11. If $\sec^{-1}x + \sec^{-1}y = \frac{\pi}{2}$ the value of $\text{cosec}^{-1}x + \text{cosec}^{-1}y$ is

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{2}$ (d) $\geq -\pi$

12. Let R be a relation defined on the set of natural numbers N as follows: $R = \{(x,y),\ 2x + y = 24\ and\ x, y \epsilon N\}$. Find the domain and range of the relation R. Also, find if R is an equivalence relation or not.

13. Show that the relation R in the set N of Natural numbers given by R = $\{(a,b): |a-b|\ is\ a\ multiple\ of\ 3\}$ is an equivalence relation.

14. Check whether the relation R in R defined by R = $\{(a,b): a \leq b^3\}$ is reflexive, symmetric, transitive.

15. Prove the relation R on the set N x N defined by (a, b) R (c, d) ⟺a+d=b+c , for all (a, b) (c, d) ϵ N x N is an equivalence relation.

16. Prove that the function f : **R**⟶ **R,** given by $f(x) = |x| + 5$, is not bijective.

17. Prove that the function f : **R**⟶ **R,** given by $f(x) = 4x^3 - 7$, is bijective

18. Prove that the Greatest Integer Function f :**R**⟶**R** given by f(x)= $[x]$, is neither one- one nor onto where $[x]$ denotes the greatest intger less than or equal to x .

19. Let f : **N** →**N** be defined by

$$f(n) = \begin{cases} \frac{n+1}{2} \; if\ n\ is\ odd \\ \frac{n}{2} \; if\ n\ is\ even \end{cases} \text{for all } n \in \mathbf{N},$$

State whether the function f is bijective.

20. Let f: **N**→ ***R*** be a function defined as f(x) = $4x^2$ +12x +15 . Check whether f:N→ R is one-one or onto.

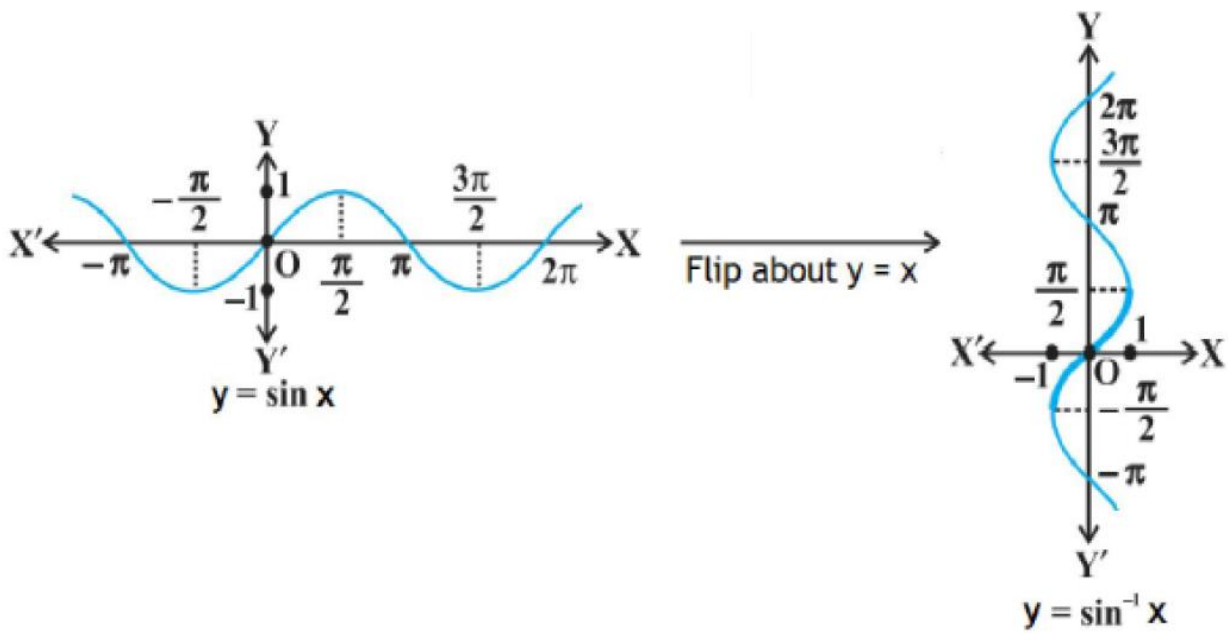
21. Consider f: **R**+→ $|[-5,\infty)$given by f(x) = $9x^2 + 6x - 5$. Check whether f:N→ R is one-one or onto.

22. Consider that f : **N** →N given by f(x) = $x^2 + x + 1$ .Check whether f:N→ N is one-one or onto.

23. Two brothers are plying their bicycles over two curved pathways.

Their teacher tells them that one of the brother is following the path of curve $y = \sin x$ while, the other one follows the path of curve $y = \sin^{-1} x$.

Refer the graphs of sine function and its inverse function, given below.



Based on the above information, answer the following :

(i) The domain and range respectively, for the sine function is given as

(A) $[-1,1], R$ (B) $R, [-1,1]$ (C) $R, [1,-1]$ (D) $[1,-1], R$

(ii) Once we restrict the domain of sine function to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then it becomes

(A) onto only (B) one-one only

(C) one-one and onto both (D) not defined

(iii) Teacher asked both the brothers, to write the range of sine inverse function other than its principal value branch. Considering the graphs shown above and strictly observing all the points shown on x, y-axes, what could be the correct answer?

(A) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (B) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ (C) $\left[-\frac{3\pi}{2}, -\frac{\pi}{2}\right]$ (D) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

(iv) What will be the principal value for $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$?

(A) $-\frac{\pi}{4}$ (B) $\frac{5\pi}{4}$ (C) $\frac{3\pi}{4}$ (D) $\frac{\pi}{4}$